LEBANESE AMERICAN UNIVERSITY
Department of Computer Science and Mathematics

Calculus III

Exam II

Fall 2014 (Nov 21, 2014)

Exam duration: 75 minutes

Name: Solutions **ID:**

QUESTION	GRADE
1. 42%	
2. 16%	
3.11 %	
4.10 %	
5. 21%	
TOTAL	

Show your work. Mention the test you are using.

1. Determine if the following series converge or diverge.

(a) $\sum_{n=1}^{\infty} \frac{(\ln n)^2}{n^{1.1}}$

7/.

$(\ln n)^2 < n^{0.01}$

$\therefore \frac{(\ln n)^2}{n^{1.1}} < \frac{n^{0.01}}{n^{1.1}} = \frac{1}{n^{1.09}}$

$\sum \frac{1}{n^{1.09}}$ p-series $p>1 \Rightarrow$ conv. $\Big\} \Rightarrow \sum \frac{(\ln n)^2}{n^{1.1}} <$ conv. series

$\Rightarrow$ conv. (by DCT)

(b) $\sum_{n=1}^{\infty} \frac{n^3}{(n^4+5n)(\ln n)^{30}}$

7/.

$\approx \sum \frac{1}{n(\ln n)^{30}}$: Logarithmic p-series $p>1$

$\Rightarrow$ conv.

(c) $\sum_{n=1}^{\infty} \frac{\cos(1/n)}{e^n}$ $\overset{LCT}{\approx} \sum \frac{1}{e^n} < \sum \frac{1}{n^2}$ (conv.) $p>1$

7/.

$\cos(1/n) \to \cos 0 = 1$

$e^n > n^2$

$\Rightarrow \sum \frac{1}{e^n} <$ conv. series $\Rightarrow$ conv. by DCT

Method 2

or $\approx \sum \left(\frac{1}{e}\right)^n$, geom series $r = \frac{1}{e} < 1 \Rightarrow$ conv.

(d) $\sum_{n=1}^{\infty} \ln\left(\frac{n+1}{3n-1}\right)$

$a_n \approx \ln(1/3) \neq 0$

$a_n \not\to 0 \Rightarrow$ $\sum a_n$ div. by n^{th} term test.

7%

(e) $\sum_{n=1}^{\infty} \sin(1/n)$ $\overset{LCT}{\approx}$ $\sum \frac{1}{n}$ div. p-series $p=1$.

9%

$\left(\frac{\sin 1/n}{1/n} \quad \xrightarrow[n\to\infty]{} 1 \right)$

(f) $\sum_{n=1}^{\infty} \frac{n^3}{n!}$

$a_n = \frac{\text{slow}}{\text{fast}} \rightarrow$ go on!!

$\rho = \lim \frac{a_{n+1}}{a_n} = \frac{(n+1)^3}{(n+1)!} \cdot \frac{n!}{n^3} \approx \frac{1}{n+1} \longrightarrow 0$

$\rho = 0 < 1 \Rightarrow \sum a_n$ conv. (Ratio test)

2. Consider the following alternating series. Determine if they converge absolutely or conditionally, or diverge.

(a) $\sum_{n=1}^{\infty} \frac{(-1)^n}{n\sqrt{n}}$

8%

Converges abs. since $\sum |\cdots| = \sum \frac{1}{n^{1.5}}$ p-series $\Rightarrow p > 1$

conv abs.

(b) $\sum_{n=1}^{\infty} (-1)^n \frac{4^n}{n^4}$

9%

$$\frac{4^n}{4^4} \longrightarrow \infty$$

$\therefore$ $a_n \not\to 0 \Rightarrow \sum (-1)^n a_n$ diverges

by n^{th} term test!!

3. Consider the series: $\sum_{n=1}^{\infty} (-3/4)^n$.

11%

(a) Evaluate its 4th partial sum, s_4

$$s_4 = a_1 + a_2 + a_3 + a_4$$

$$-\frac{3}{4} + \left(\frac{3}{4}\right)^2 + \left(\frac{-3}{4}\right)^3 + \left(\frac{3}{4}\right)^4$$

(b) Evaluate its limit

$$= \frac{\text{first term}}{1 - r} = \frac{-3/4}{1 + 3/4} = \frac{-3/4}{5/4}$$

$r = -\frac{3}{4}$

$|r| < 1 \Rightarrow$ conv.

$\longrightarrow -\frac{3}{5}$

(c) Estimate the error involved when you approximate the series to be s_4

$$|E| \le a_5 = \left|\left(-\frac{3}{4}\right)^5\right|$$

4. Find the values of x for which the following series converges: $\sum_{n=1}^{\infty} \frac{(2x-1)^n}{n^3}$

10%

$$\rho = \lim \left|\frac{(2x-1)^n}{n^3}\right|^{1/n} \longrightarrow \frac{|2x-1|}{n^{3/n}} \longrightarrow |2x-1|$$

$n^{3/n} \to 1$

$\therefore$ Need $\rho < 1 \Rightarrow |2x-1| < 1$

$\Rightarrow$

$$-1 < 2x-1 < 1$$

$$0 < 2x < 2$$

$$\boxed{0 < x < 1}$$

(1) check $x=0$ Series yields: $\sum \frac{(-1)^n}{n^3}$

conv. ✓

(2) check $x=1$ Series yields: $\sum \frac{1}{n^3}$

also conv.

$\therefore$ Series conv. for $\boxed{0 \le x \le 1}$

5. Find the MacLaurin series for the following functions. (Use results that have already been found in class)

(a) $f(x) = xe^{(x^2)}$

21%

$$e^t = 1 + t + \frac{t^2}{2!} + \frac{t^3}{3!} + \cdots$$

$$e^{x^2} = 1 + x^2 + \frac{x^4}{2!} + \frac{x^6}{3!} + \cdots$$

$$xe^{x^2} = x + x^3 + \frac{x^5}{2!} + \frac{x^7}{3!} + \cdots$$

(b) $f(x) = \dfrac{1}{(x+1)^2}$, where $|x| < 1$.

$$\left(\frac{1}{x+1}\right)^2 =$$

$$\frac{1}{x+1} = 1 - x + x^2 - x^3 + x^4 - x^5 + \cdots$$

$$\left(\frac{1}{x+1}\right)' = \frac{-1}{(x+1)^2} = -1 + 2x - 3x^2 + 4x^3$$

$$\therefore \left(\frac{1}{x+1}\right)^2 = 1 - 2x + 3x^2 - 4x^3 + \cdots$$

(c) $f(x) = \ln(1 + x^2)$. where $|x| < 10$.Find its MacLaurin series in 2 ways. (both times by using a ready template)

I

$$\ln(1 + x^2) = \ln(1+t) = \int \frac{dt}{1+t} = \int 1 - t + t^2 - t^3 + t^4 - t^5 + \cdots$$

$t = x^2$

$$= t - \frac{t^2}{2} + \frac{t^3}{3} - \frac{t^4}{4} + \frac{t^5}{5} \cdots$$

$$\therefore \ln(1 + x^2) = x^2 - \frac{x^4}{2} + \frac{x^6}{3} - \frac{x^8}{4} + \frac{x^{10}}{5} - \cdots$$

II

$$\ln(1+x^2) = \int \frac{2x\,dx}{1+x^2} = \int 2x(1 - x^2 + x^4 - x^6 + x^8 - x^{10} + \cdots)\,dx.$$

$$= \int 2x - 2x^3 + 2x^5 - 2x^7 + 2x^9 - 2x^{11} + \cdots \,dx$$

$$= 2\left[\frac{x^2}{2} - \frac{x^4}{4} + \frac{x^6}{6} - \frac{x^8}{8} + \cdots\right]$$

$$= x^2 - \frac{x^4}{2} + \frac{x^6}{3} - \frac{x^8}{4} + \cdots$$